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Estimation of mass fraction of residual gases from cylinder pressure data and its application to modeling for SI engine

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Abstract
To have good performance with high efficiency and lower pollutants in internal combustion engine mass fraction of residual gas is of high importance. The purpose of this study is to introduce a novel method to calculate the mass fraction of residual gas from cylinder pressure data. In this method the fraction of the residual gases is calculated by combining the first law of thermodynamics and ideal gas law. To calculate heat release rate in combustion flame speed method has been used. Relation of flame laminar speed and temperature and pressure parameters and air fuel ratio to mass fraction burned were used to do the calculation. Finally the results from algorithm calculation of mass fraction for residual gases have been compared with the results obtained from the ideal model of calculation of mass fraction for the residual gases in the same engine's speed.

Keywords: SI Engine, Residual Gas Fraction, Flame Speed, Heat Release Rate.

1 Introduction

The purpose for internal combustion engines is to produce mechanical power from chemical energy of fuel. The energy in this type of engine is released by the burning or oxidation within the engine. Before combustion mixture of fuel, air and gases and after combustion the products of combustion are the working fluid for the engines of this type. Combustion occurrence in the power production section of the engines has made its design and performance characteristics distinct from other engines. During the past three decades, new parameters were introduced which exerted a considerable impact on engine performance and design. These new parameters were needed at the first place for the control of the vehicle share of air pollution and at the second place for the reduction of fuel consumption [1].

Single-zone models which are generally used in modeling of the performance of internal combustion engines for the achievement of optimum conditions of performance and for the examination of various engine performance parameters have much application. The basic assumption for the modeling of this type is homogeneity of charges and mixing of gases inside the cylinder. To model the various processes thermodynamic analysis of a Single-zone has been used regardless of leakage into and out of cylinder.

The proposed method for predicting combustion is flame speed method. The relation of flame laminar speed and temperature and pressure parameters and air fuel ratio to mass fraction burned were used to do the calculation. To calculating mass fraction of residual gases a novel method has been introduced. Combining the first law of thermodynamics, ideal gas law and qualitative pre-assumptions will result in desirable answers. The above said models and qualitative pre-assumptions have been

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applied to a single cylinder engine and constant compression ratio and then the results from single cylinder engine four stroke 392cc volume are evaluated and compared with the mass fraction of the residual gases obtained from the two methods on the engine performance parameters.

2 Data analysis

2.1 Estimation of mass fraction of residual gases by ideal four stroke cycle analysis

In Ideal cycle analysis intake and ideal exhaust process are assumed to be of constant pressure and adiabatic (fig(1)). When the intake and ideal exhaust process in the cycle calculations are given two additional equations: exhaust energy equation and intake energy equation. Two unknown parameters in these equations are residual mass fraction (f) and gas temperature at the end of intake (T_1). Since solving two algebraic equations simultaneously is difficult, repetition procedure (iteration) has been employed.

Since (T_1) is not an independent parameter and depends on the residual mass fraction , (f) and temperature of these gases (T_e) and if the values of (T_e) and (f) assumed to be specified, calculations of all cycles are done and by repeating the calculations (iteration) values (f) and (T_e) will be improved (T_i the temperature of incoming charge and q_{in} released heat per unit mass of fuel mixture and compression ratio is r)[2].

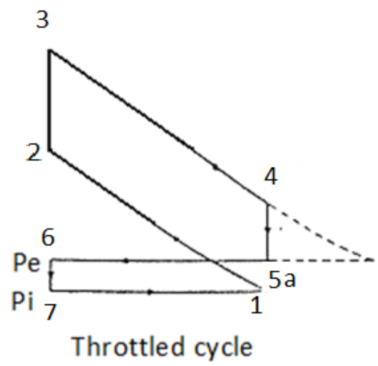

Fig. 1 Exhaust and ideal intake for four strokes engines

6, i -1: Intake process

$$p_1 = p_i$$

$$T_1 = (1-f)T_i + fT_e\left[1-(1-\frac{P_i}{P_e})\left(\frac{\gamma-1}{\gamma}\right)\right]$$

1-2: Isentropic compression process

$$T_2 = T_1 r^{(\gamma-1)}$$

$$p_2 = p_1(V_1/V_2)^{\gamma} = p_1 r^{\gamma}$$

2-3: Energy increase in constant volume

$$p_3 = p_2(T_3/T_2) \qquad T_3 = T_2 + q_{in}(1-f)/c_v$$

3-4: Isentropic expansion process

$$T_4 = T_3(1/r)^{(\gamma-1)}$$

$$p_4 = p_3(1/r)^{\gamma}$$

4-5: Isentropic lowdown

$$p_5 = p_e$$

$$T_5 = T_4(p_4 / p_e)^{(1-\gamma)/\gamma}$$

5-6: The process of constant pressure exhaust

$$T_e = T_5 \qquad p_6 = p_5 = p_e$$

$$f = 1/r(p_6 / p_4)^{1/\gamma}$$

2.2 Flame speed method in modeling of combustion process

In the single zone model it is assumed that the pressure, temperature and charge composition inside the cylinder are uniform. In this model there is no distinction between burnt and unburned gases and the assumption is made that the charge is homogeneous. The advantage of single zone model is that heat transfer and gas leakage to the inside and outside of the cylinders makes them easier to express. If the first law of thermodynamics for a closed system (regardless of crevice [3] and [2]) is written will lead to:

$$\frac{dP}{d\theta} = -\gamma\frac{P}{V}\frac{dV}{d\theta} + \frac{\gamma-1}{V}\left(\frac{dQ_{tot}}{d\theta}\right) \tag{1}$$

$$\frac{dQ_{tot}}{d\theta} = \frac{dQ_{hr}}{d\theta} - \frac{dQ_{ht}}{d\theta} \tag{2}$$

and dQ_{hr} represent chemical energy released by combustion, and dQ_{ht} will be the heat transfer to combustion chamber walls.

In the analyses of the single zone for the study of combustion process one model is always needed. One of the methods for modeling the combustion process in the cylinder chamber is flame speed method. In this method laminar flame speed is related to the mass fraction burned and energy released and the cylinder pressure by properties such as temperature, pressure and air fuel ratio.

Since the actual flame speed in an SI engine depends on the turbulent intensity, turbulent model should also be taken into account. This model is a simple turbulent model, where turbulence intensity is assumed to be a function of engine speed [3]. The laminar Flame speed of a premixed gasoline/air flame will steadily increase with temperature and decreases uniformly with pressure. This behavior with the following empirical relationship is turned into model:

$$S_L = S_{L,O}\left(\frac{T_u}{T_o}\right)^{\alpha}\left(\frac{p}{p_o}\right)^{\beta} \tag{3}$$

In this relationship $S_{L,0}$ is the laminar flame speed at $T_0 = 298K$ and $P_0 = 1atm$. The parameters α , β depend slightly on the equivalence ratio. At normal pressure and temperature, $S_{L,0}$ can be obtained from the following relation:

$$S_{L,0} = B_m + B_\varphi(\varphi - \varphi_m)^2 \tag{4}$$

In this relationship, φ_m is the equivalence ratio in which the maximum burning velocity (B_m) occurs. Rapid burn angle with inlet pressure and constant equivalence ratio will almost change with engine speed as an exponential function:

$$\Delta\alpha_b \cong N^{0.37} \tag{5}$$

Since the turbulent flow characteristics will change with engine speed and depends on the engine geometry, the exponent, N is unknown and is shown with η that is obtained from the experimental results. Thus:

$$\Delta\alpha_b \cong KN^{\eta} \tag{6}$$

During the rapid burn phase, it is assumed that the burn rate is constant. It is due to the fact that the combustion chamber is enclosed and the spherical flame front will be cut off by the combustion chamber. Figure (2) shows the cross section of the cylinder combustion chamber and progression of front flame from a centrally located spark plug and it is seen that the area of the flame front stays constant as long as it reaches the end of the combustion. A more realistic model is that the area of the flame during the rapid burn phase is constant and this means that the flame volume grows with a constant rate. By the above assumptions the following relationship for the burning rate will be obtained based on temperature, pressure and engine speed:

$$\frac{dm_f}{d\alpha} = bp^{1+\alpha}T^{\beta-1}N^{-\eta}.AFR^{-1} \tag{7}$$

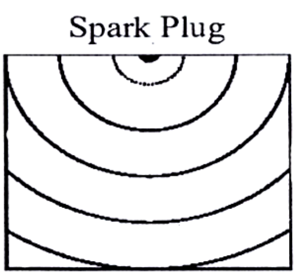

Fig. 2 Two-dimensional illustration of flame propagation from the spark plug

Which be equals to:

$$b = \frac{60 A_f U_{0,0}}{2\pi R T_0^{\alpha} P_0^{\beta} N_0^{1-\eta}} \tag{8}$$

Where A_f, Flame front area is constant and $U_{\circ,\circ}$ is laminar flame speed at a normal temperature and pressure.

2.3 Estimation of the Residual Gas fraction by Cylinder Pressure

The residual gas fraction is an important parameter to have good performance with high efficiency and lower pollutants in internal combustion engine. The purpose is to introduce an algorithm to calculate the mass fraction of residual gases [4]. Total mass inside the cylinder includes fuel, air and residual gases.

$$m_{tot} = m_{fuel} + m_{air} + m_{rg} \tag{9}$$

If the amount of total mass, the air and fuel inside the cylinder are known mass fraction of the residual gases can be calculated. In this algorithm the cylinder pressure is defined as input.

Charge efficiency, η_c is defined as a ratio between $m_{\min}$ and m_{tot}. m_{tot} is the total mass in the cylinder and $m_{\min}$ is the theoretical minimum mass of air and fuel that is required for releasing energy from fuel:

$$\eta_c = \frac{m_{\min}}{m_{tot}} \tag{10}$$

Minimum mass is calculated from the following relationship:

$$m_{\min} = \frac{Q_{fuel}}{Q_{LHV}}(L_{st} + 1) \tag{11}$$

The actual value of the cylinder air can be described as follows:

$$m_{air} = \lambda L_{st} m_{fuel} \tag{12}$$

The amount of residual gases within the cylinder is expressed as part of the total mass:

$$m_{rg} = x_{rg} m_{tot} \tag{13}$$

The amount of fuel in the minimum mass is defined by completeness of combustion, COC :

$$m_{fuel} = \frac{1}{coc}\frac{Q_{fuel}}{Q_{LHV}} \tag{14}$$

If the Charge is diluted the value of coc is high and about 96 to 98%. In this study, the value is 96%. By combining the equations from (11), (12), (13) and (14) the total mass is calculated as follows:

$$m_{tot} = \frac{1}{1-x_{rg}}\left(\frac{1}{coc}\frac{Q_{fuel}}{Q_{LHV}}(\lambda L_{st}+1)\right) \tag{15}$$

If equations (11) and (15) are inserted in the definition of charge efficiency the following equation is obtained for mass fraction of residual gases.

$$x_{rg} = 1 - \frac{\eta_c}{coc}\left(\frac{\lambda L_{st}+1}{L_{st}+1}\right) \tag{16}$$

In experimental research the value of coc is assumed constant but the value is measured by the sensor. Charge efficiency, η_C is calculated as follows:

$$\eta_C = \frac{m_{\min}}{m_{tot}} = \frac{m_{\min}}{\frac{P_1V_1}{R_1T_1}} \tag{17}$$

The $m_{\min}$ is calculated from equation (11) and m_{tot} is calculated by the application of the ideal gas law at $80^\circ BTDC$. The unknown parameter in this relationship is T_1.

2.4 Temperature and Total Mass in the Cylinder

By using the first law of thermodynamics it is possible to calculate temperature and total mass of the inside of the cylinder. The internal energy difference between $80^\circ BTDC$ and 50% the spot where the mass is burnt includes the heat released by burning fuel, Q_{fuel}, mechanical work, W ,and heat loss into cylinder wall, Q_{wall}. Fuel energy can be described by Charge efficiency and the total mass:

$$Q_{fuel} = \frac{m_{tot}\eta_c Q_{LHV}}{L_{st}+1} \tag{18}$$

The mechanical work is calculated by integrated from the volume at $80^\circ BTDC$, (V_1) to the volume at $50\% mfb$ position, (V_2).

$$W = \int_{V_1}^{V_2} PdV \tag{19}$$

The heat losses, Q_{wall} , as a function of engine speed and combustion energy and relationships can be expressed as:

$$Q_{wall} = 2((\frac{-0.2161}{N} - 0.0049) \times 2Q_{fuel} + \frac{-90.78}{N} - 3.91) \quad (20)$$

Then the internal energy at $50\%\, mfb$ is calculated as follows:

$$U_{50} = U_1 + Q_{fuel} - Q_{wall} - W \quad (21)$$

Ultimately the temperature is calculated as:

$$T_{50} = \frac{U_{50}}{\bar{C}_{v\,50} m_{tot}} \quad (22)$$

To calculate temperature and mass, heat capacity, C_v, and gas constant, R ,are required. These parameters are dependent on the composition of gases and cylinder temperature. Since the calculation of the temperature and total mass requires these parameters and the parameters are dependent on the temperature, the algorithm is iterated to achieve better result.

Gas constant only depends on the composition of gas, but the heat capacity depends on both temperature and the composition of gas.
Linear approximations of $\bar{C}_v$ are as follows:

$$C_{v_{rg\,50}} = .2426 T_{50} + 615.44 \;\; [j / Kg.K] \quad (23)$$

$$C_{v_{fg50}} = .00482 T_{50} + 738.4 \; [j / Kg.K] \quad (24)$$

$$C_{v_{50}} = (1 - x_{rg}) \; C_{v_{fg}} + x_{rg} \;\; C_{v_{rg}} \;\; [j / Kg.K] \quad (25)$$

Gas constant for the residual gases and fresh charge are equal to:

$$R_{fg} = 274 \, [\, {}^{j}\!/_{Kg.K} \,] \quad (26)$$

$$R_{rg} = 287 \, [\, {}^{j}\!/_{Kg.K} \,] \quad (27)$$

During iteration when a new T_{50} and x_{rg} obtained heat capacity and gas constant will change (Figure (3)).

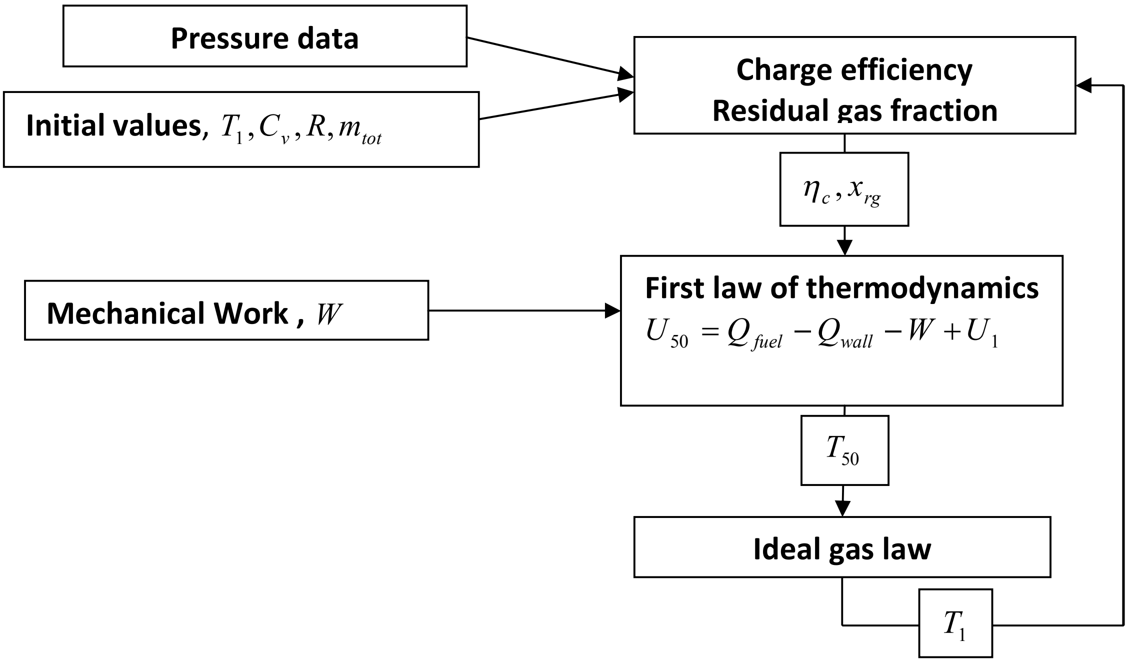

Fig. 3 Flowchart over the algorithm

2.5 Residual Gas Mass

This algorithm does not often result in the correct temperature value. The mass of residual gas is then estimated by using the exhaust gas temperature and volume at exhaust valve closing.

$$m_{rg} = \frac{P_{cvc} V_{cvc}}{R_{rg} T_{exhaust}} \tag{28}$$

Mass of residual gases has two different applications: the direct calculation of the mass fraction of residual gas by dividing it by the total mass. It does not lead to the desirable results. The second one work betters. A new total mass is calculated by the last repetition (Fig (4)):

$$m_{tot} = m_{fuel} + m_{air} + m_{rg} = m_{tot}(1 - x_{rg}) + m_{rg} \tag{29}$$

Where Mass of residual gases from equation (28) is calculated and finally mass fraction of residual gas is equal to:

$$x_{rg} = \frac{m_{rg}}{m_{tot}} = \frac{m_{rg}}{m_{fuel}(1 + \lambda L_{st}) + m_{rg}} \tag{30}$$

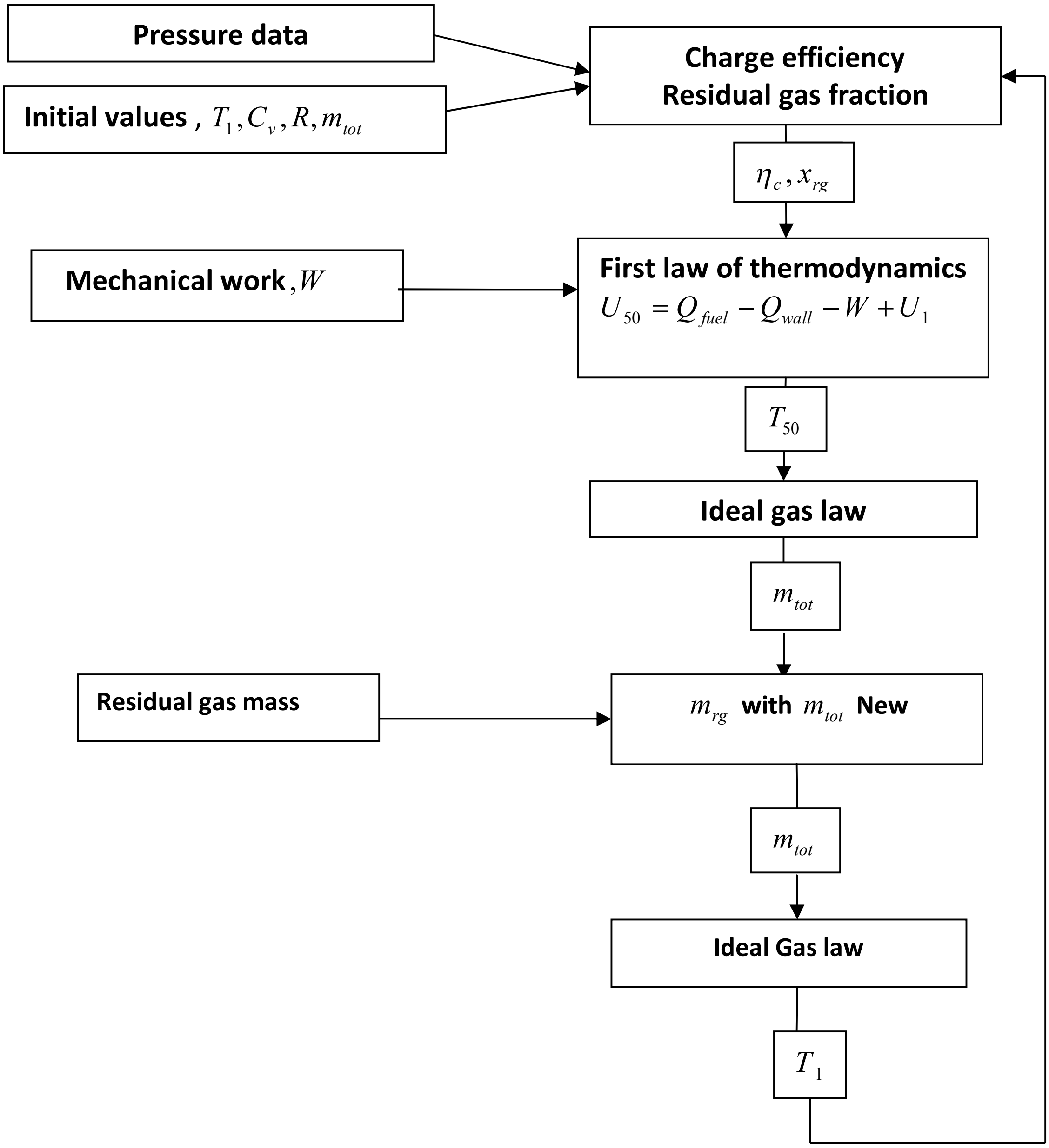

Fig. 4 flowchart over the algorithm with m_{rg}

3 Data Analysis and the Results

The above methods have been applied to four stroke single cylinder engine on a 392cc volume and fixed compression ratio. Its characteristics have been given in Table 1. Connecting rod length to crank radius is $R = 3.5$. The mass of fuel that is used for the calculation of fuel energy released per cycle has been obtained by the definition of volumetric efficiency and equivalence fuel air ratio and the amount of mass fraction of residual gases. The value of wall mean temperature (average temperature of piston, cylinder head and cylinder walls) which is inserted in the heat loss relationship is assumed constant $400K$ [2]. In this article mass fraction of residual gases has been calculated by the two methods in different speeds by keeping other parameters constant.

Equivalence fuel air ratio equal to 1.05, the ignition time is 30° before TDC and the combustion duration, θ_d is 60° which ranges from the proposed limit 50° to 80° of the SI engines[2]. Overall pressure of the exhaust posses is from $1\,atm$ to $1.05\ atm$.In this article the exhaust pressure has been selected to be $1.05\ atm$.Inlet Charge pressure on the bases of volumetric efficiency and gasoline (C_7H_{17}), engine speed and ideal gas law at the onset of intake has been calculated. Gasoline (C_7H_{17}) has been selected for Fuel. Flame Speed method has been used for combustion modeling. It is worth noting that the temperature of unburned gas has been constant and 500K which are at the proposed range of 300-500 k. The other point is that in the calculation of the laminar flame speed, values of φ_m, B_m and B_φ related to the fuel (C_7H_{17}), which are cited in the references [1].

The purpose of this article is to calculate the exact amount or mass fraction of residual gases by the ideal methods and the use of pressure data and finally the results on figures of indicator effective pressure and brake thermal efficiency are examined. In this method the value of f from ideal analysis is first calculated by an initial guess in a known speed. With the new value of f, input temperature and the amount of fuel has been calculated. Then the pressure differential equation is solved by the flame speed method in estimation of the combustion. Pressure data generated from its calculation are used as input for the algorithm to calculate the mass fraction.

By a computer program written for this purpose the amount of f is achieved and its value is used as a guessed value in ideal analysis and new value is achieved by ideal analysis .By this new value the pressure differential equation can again be calculated and the data again inserted as the input in the algorithm and the next value of f is calculated with the value obtained from the previous step. This will be repeated a few times to converge to a constant value.

And finally pressure differential equation is solved by the obtained amount of mass fraction of residual gases and engine performance parameters at different speed are calculated.

The main input of this algorithm is pressure that should be measured by the sensors placed inside the cylinder. However, due to the lack of facilities, the calculated pressures from flame speed method are used in the algorithm.

Figure (5) shows the comparison of mass fraction of residual gases by the two ways of the ideal method and the cylinder pressure method. As it is observed calculated points in one speed by the two methods differs nearly up to 5%.

As is clear in the next Graphs the difference in engine performance parameters figures (6) and (7) and cylinder pressure difference in figures (8) and (9) is up to 5% at most states. Figures (6) and (7) show speed effect on mean effective pressure and indicator thermal efficiency. As can be seen in the figures, with engine speed increase, the mean effective pressure and thermal efficiency are increased. The reason is heat loss reduction with engine speed increase. As a result of heat loss reduction pressure inside the cylinder increases and thus transferred work to the piston increases. As can be seen in the figures in high speeds indicator thermal efficiency and imep values change due to changes in volumetric efficiency.

This method leads to desirable results by combining the first law of thermodynamics, ideal gas law and pre- assumption.

The advantages of this new method are that it is not mathematically complex and that by few repetitions the desirable results can be achieved and that total thermal losses in the whole system cycles and changes in working fluid properties are also considered. What seems important is the fact that if the temperature of exhaust gases with a pressure cylinder enters the calculations more accurate solutions can be obtained.

4 Appendix

Specific heat capacity dependence with temperature can be estimated by the following the relationship [6]:

$$\gamma = 1.4 - 7.18\times10^{-5}T$$

Woschni relationship for Heat transfer coefficient equals [2]:

$$h_g(\theta) = 3.26 b(m)^{-0.2}\,U\left(m/_s\right)^{0.8}.P(Kpa)^{0.8}.T(K)^{-0.55}$$

And the unity of h_g is $w/m^2.k$

Table 1 Engine Specifications
Single cylinder engine
P8161 BRIGSS AND STRATION

Type	4 stroke , air cooled petrol engine
Capacity	392 cc (77.8 mm bore , 82.5 mm stroke)
Compression Ratio	9:1
Maximum Power	7.46 KW at 3600 rpm
Maximum Torque	22.7 N.m at 2400 rpm

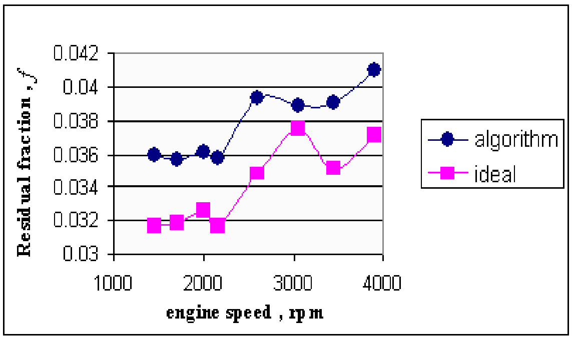

Fig. 5 The comparison of mass fraction of residual gases by the two ways of the ideal Otto cycle and algorithm

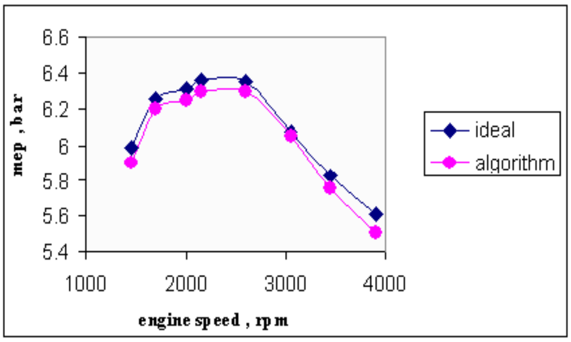

Fig. 6 The effect of mass fraction of residual gasses calculated by the two methods of ideal Otto cycle and algorithm on indicator effective pressure

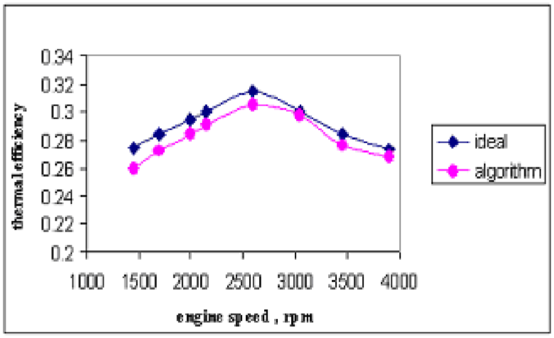

Fig. 7 The effect of mass fraction of residual gasses calculated by the two methods of ideal Otto cycle and algorithm on indicator heat efficiency

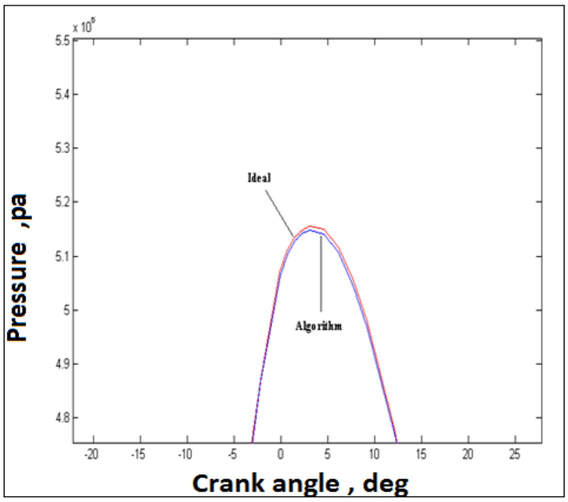

Fig. 8 The effect of mass fraction of residual gasses calculated by the two methods of the ideal Otto and algorithm at 1450 rpm

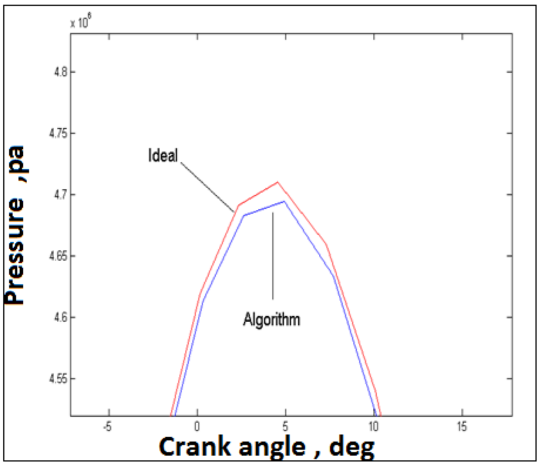

Fig. 9 The effect of mass fraction of residual gasses calculated by the two methods of ideal Otto cycle and algorithm at 3050 rpm

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